The values of the relative boundary-layer thickness, displacement thickness, momentum thickness, and local tangential stress on the plate in relation to α are given in Table 2.

It should be noted that the obtained solutions include, as a special case of the considered problem, the solution for the boundary layer of a Newtonian fluid. For this we must put $s = 0$.

NOTATION

x, y, z, Cartesian coordinates; u, v, velocity vector components; w, microrotation vector component; ν, ν_1, ν_2, coefficients of shear, rotational, and couple viscosity; $\mu = \nu/\rho$, dynamic coefficient of shear viscosity; ρ, density; w_0, microtation on plate; U_∞, velocity of incident flow; $s = \nu_1/\nu$, $m = j\nu_2/\nu$, dimensionless rheological parameters of fluid model; j, microinertia; A, B, α, constants; Ψ, stream function; η, self-similar variable; f, φ, dimensionless stream and microrotation functions; t_{xy}, t_{yx}, stress tensor components; m_{xy}, m_{yx}, couple stress tensor components; τ_0, local tangential stress on plate; δ, relative boundary-layer thickness; δ^*, displacement thickness; δ^{**}, momentum thickness.

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HEAT TRANSFER IN A STRETCHED BICOMPONENT FILM

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The temperature distribution in a bicomponent film during nonisothermal stretching is discussed without allowance for dissipative heating.

Bicomponent films (laminates) are increasingly used, because such films are of particularly good performance, e.g., in the production of crimped materials, where the crimping arises from differences in elasticity of the components. A polymer characteristically has very marked temperature dependence of the elastic parameters, so the working temperature is a major parameter in production of crimped bicomponent material. Appropriate thermal conditions can be provided in pulling such films in order to control the crimping [1, 2].

Here we consider the temperature pattern in such a film during nonisothermal stretching.

Figure 1 shows the drawing scheme.

We use an immobile Euler coordinate system with the x axis coincident with the direction of motion of the film, while the y axis is perpendicular to that direction, and the origin and the x axis are equidistant from the outer surfaces of the film.

We assume a linear distribution for the axial velocity of the film in the drawing zone [3-5]:

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$$v_x = v_0 + \Gamma x, \tag{1}$$

where $\Gamma = (v_f - v_0)/L$ is the mean velocity gradient.

We use the integral equation of continuity to get the following equations for the surfaces of the components:

$$\beta_1 = \frac{-\delta_0 v_0}{2(v_0 + \Gamma x)}, \quad \beta_2 = \frac{\delta_0 v}{2(v_0 + \Gamma x)},$$

$$\beta_c = \frac{\delta_0 (q - 0.5) v_0}{v_0 + \Gamma x},$$

where $q = \delta_1/\delta_0$ is the relative thickness of the first component. The equation of continuity for the planar case takes the form

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0. \tag{2}$$

As the drawing involves two polymer films differing in thermophysical parameters and the components are in contact, we get a system of two energy equations (with dissipative heating neglected):

$$\begin{aligned} (v_0 + \Gamma x)\frac{\partial T_1}{\partial x} - \Gamma y \frac{\partial T_1}{\partial y} &= a_1 \frac{\partial^2 T_1}{\partial y^2}, \\ (v_0 + \Gamma x)\frac{\partial T_2}{\partial x} - \Gamma y \frac{\partial T_2}{\partial y} &= a_2 \frac{\partial^2 T_2}{\partial y^2}. \end{aligned} \tag{3}$$

The temperature distribution in the film is found by solving (3) subject to the following boundary conditions:

$$x = 0, \quad T_1 = \psi_1(y), \tag{4}$$

$$x = 0, \quad T_2 = \psi_2(y), \tag{5}$$

$$y = \beta_1, \quad -\lambda_1 \frac{\partial T_1}{\partial y} + \alpha_1 (T_1 - T_{c_1}) = 0, \tag{6}$$

$$y = \beta_2, \quad \lambda_2 \frac{\partial T_2}{\partial y} + \alpha_2 (T_2 - T_{c_2}) = 0, \tag{7}$$

$$y = \beta_c, \quad T_1 = T_2, \tag{8}$$

$$y = \beta_c, \quad \lambda_1 \frac{\partial T_1}{\partial y} = \lambda_2 \frac{\partial T_2}{\partial y}. \tag{9}$$

The functions $\psi_1(y)$ and $\psi_2(y)$ satisfy the Dirichlet conditions and also the conjugation conditions of (8) and (9).

We make the substitutions

$$t_1 = T_1 - T_{c_1}, \quad t_2 = T_2 - T_{c_2}.$$

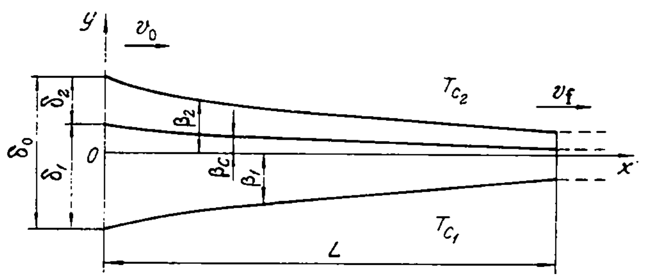

Fig. 1. Scheme for drawing of a bicomponent film.

Then (3) takes the form

$$(v_0 + \Gamma x)\frac{\partial t_1}{\partial x} - \Gamma y \frac{\partial t_1}{\partial y} = a_1 \frac{\partial^2 t_1}{\partial y^2},$$
$$(v_0 + \Gamma x)\frac{\partial t_2}{\partial x} - \Gamma y \frac{\partial t_2}{\partial y} = a_2 \frac{\partial^2 t_2}{\partial y^2}. \quad (10)$$

The boundary conditions are correspondingly put as

$$x = 0, \quad t_1 = \psi_1(y) - T_{c_1}, \quad (11)$$
$$x = 0, \quad t_2 = \psi_2(y) - T_{c_2}, \quad (12)$$
$$y = \beta_1, \quad -\lambda_1 \frac{\partial t_1}{\partial y} + \alpha_1 t_1 = 0, \quad (13)$$
$$y = \beta_2, \quad \lambda_2 \frac{\partial t_2}{\partial y} + \alpha_2 t_2 = 0, \quad (14)$$
$$y = \beta_c, \quad t_1 + T_{c_1} = t_2 + T_{c_2}, \quad (15)$$
$$y = \beta_c, \quad \lambda_1 \frac{\partial t_1}{\partial y} = \lambda_2 \frac{\partial t_2}{\partial y}. \quad (16)$$

System (10) is solved by separating the variables; let

$$t_1 = f_1(x)\varphi_1(y), \quad t_2 = f_2(x)\varphi_2(y). \quad (17)$$

System (10) takes the following form after separating the variables:

$$(v_0 + \Gamma x)\frac{f_1'}{f_1} = a_1 \frac{\varphi_1''}{\varphi_1} + \Gamma y \frac{\varphi_1'}{\varphi_1} = S_1,$$
$$(v_0 + \Gamma x)\frac{f_2'}{f_2} = a_2 \frac{\varphi_2''}{\varphi_2} + \Gamma y \frac{\varphi_2'}{\varphi_2} = S_2, \quad (18)$$

where S_1 and S_2 are eigenvalues; the solutions for the functions $f_1(x)$ and $f_2(x)$ take the form

$$f_1(x) = (v_0 + \Gamma x)^{\frac{S_1}{\Gamma}}, \qquad f_2(x) = (v_0 + \Gamma x)^{\frac{S_2}{\Gamma}}.$$

We reduce (18) to a Weber equation [6] in order to define $\varphi_1(y)$ and $\varphi_2(y)$.

The solution to (18) is

$$t_1 = C_1(v_0 + \Gamma x)^{\frac{S_1}{\Gamma}}\left[F_1(y, S_1) + C_2 iy\sqrt{\frac{\Gamma}{a_1}}F_2(y, S_1)\right],$$
$$t_2 = C_3(v_0 + \Gamma x)^{\frac{S_2}{\Gamma}}\left[F_1(y, S_2) + C_4 iy\sqrt{\frac{\Gamma}{a_2}}F_2(y, S_2)\right], \quad (19)$$

where

$$F_1(y, S_k) = 1 + \sum_{\nu=1}^{\infty} \frac{\left(-\frac{S_k}{\Gamma}\right)\left(2 - \frac{S_k}{\Gamma}\right)\cdots\left(2\nu - 2 - \frac{S_k}{\Gamma}\right)}{(2\nu)!}(-1)^{\nu}\left(\sqrt{\frac{\Gamma}{a_k}}y\right)^{2\nu},$$
$$F_2(y, S_k) = 1 + \sum_{\nu=1}^{\infty} \frac{\left(1 - \frac{S_k}{\Gamma}\right)\left(3 - \frac{S_k}{\Gamma}\right)\cdots\left(2\nu - 1 - \frac{S_k}{\Gamma}\right)}{(2\nu + 1)!}(-1)^{\nu}\left(\sqrt{\frac{\Gamma}{a_k}}y\right)^{2\nu};$$

where k = 1 or 2 is the number of a component, and F_1 and F_2 are Pachhammer functions or degenerate hypergeometric functions [6].

The unknowns C_1-C_4 and the eigenvalues S_1 and S_2 are derived from (11)-(16).

From (13) we get C_2:

$$C_2 = \frac{-[\alpha_1 F_1(\beta_1, S_1) - \lambda_1 F_1'(\beta_1, S_1)]}{i\left[-\lambda_1 \sqrt{\frac{\Gamma}{a_1}} F_2(\beta_1, S_1) - \lambda_1 \sqrt{\frac{\Gamma}{a_1}} \beta_1 F_2'(\beta_1, S_1) + \alpha_1\beta_1 \sqrt{\frac{\Gamma}{a_1}} F_2(\beta_1, S_1)\right]}, \quad (20)$$

Similarly from (14) we have

$$C_4 = \frac{-[\alpha_2 F_1(\beta_2, S_2) + \lambda_2 F_1'(\beta_2, S_2)]}{i\left[\lambda_2 \sqrt{\frac{\Gamma}{a_2}} F_2(\beta_2, S_2) + \lambda_2 \sqrt{\frac{\Gamma}{a_2}} \beta_2 F_2'(\beta_2, S_2) + \alpha_2\beta_2 \sqrt{\frac{\Gamma}{a_2}} F_2(\beta_2, S_2)\right]}, \quad (21)$$

where

$$F_1'(\beta_k, S_k) = \frac{\partial F_1(\beta_k, S_k)}{\partial y}, \quad F_2'(\beta_k, S_k) = \frac{\partial F_2(\beta_k, S_k)}{\partial y},$$

and k = 1, 2 are the numbers of the components. To derive C_1 we use (11):

$$C_1 Z_1(y, S_1) = \psi_1(y) - T_{c_1}, \quad (22)$$

where

$$Z_1(y, S_1) = v_0^{\frac{S_1}{\Gamma}}\left[F_1(y, S_1) + C_2 iy \sqrt{\frac{\Gamma}{a_1}} F_2(y, S_1)\right].$$

This particular solution does not satisfy the initial condition, so we write the general solution as an infinite sum of particular solutions:

$$\sum_{n=1}^{\infty} C_1 Z_{1n}(y, S_{1n}) = \psi_1(y) - T_{c_1}. \quad (23)$$

The system of functions $Z_{1n}(y, S_{1n})$ is complete and orthogonal; we multiply both sides of (23) by $Z_{1m}(y, S_{1m})dy$ and take the integral with limits β_1 and β_C [7]:

$$\sum_{n=1}^{\infty} C_1 \int_{\beta_1}^{\beta_c} Z_{1n}(y, S_{1n}) Z_{1m}(y, S_{1m})\, dy = \int_{\beta_1}^{\beta_c} [\psi_1(y) - T_{c_1}] Z_{1m}(y, S_{1m})\, dy,$$

The orthogonality implies that the integral on the left is zero for $m \neq n$, and therefore

$$C_1 = \frac{\int_{\beta_1}^{\beta_c} [\psi_1(y) - T_{c_1}] Z_{1n}(y, S_{1n})\, dy}{\int_{\beta_1}^{\beta_c} Z_{1n}^2(y, S_{1n})\, dy}. \quad (24)$$

Similarly we have from (12) that

$$C_3 = \frac{\int_{\beta_c}^{\beta_2} [\psi_2(y) - T_{c_2}] Z_{2n}(y, S_{2n})\, dy}{\int_{\beta_c}^{\beta_2} Z_{2n}^2(y, S_{2n})\, dy}, \quad (25)$$

where

$$Z_{2n}(y, S_{2n}) = v_0^{\frac{S_{2n}}{\Gamma}}\left[F_1(y, S_{2n}) + C_4 iy \sqrt{\frac{\Gamma}{a_2}} F_2(y, S_{2n})\right].$$

The eigenvalues S_{1n} and S_{2n} are determined as the roots of a system of transcendental equations derived from (15) and (16):

$$T_{c_1} + C_1 (v_0 + \Gamma x)^{\frac{S_{1n}}{\Gamma}} \left[F_1(\beta_c, S_{1n}) + C_2 i \beta_c \sqrt{\frac{\Gamma}{a_1}} F_2(\beta_c, S_{1n}) \right] =$$
$$= T_{c_2} + C_3 (v_0 + \Gamma x)^{\frac{S_{2n}}{\Gamma}} \left[F_1(\beta_c, S_{2n}) + C_4 i \beta_c \sqrt{\frac{\Gamma}{a_2}} F_2(\beta_c, S_{2n}) \right], \tag{26}$$
$$\frac{\lambda_1 C_1}{\lambda_2 C_3} (v_0 + \Gamma x)^{\frac{S_{1n} - S_{2n}}{\Gamma}} \left\{ F_1'(\beta_c, S_{1n}) + C_2 i \sqrt{\frac{\Gamma}{a_1}} \left[F_2(\beta_c, S_{1n}) + \beta_c F_2'(\beta_c, S_{1n}) \right] \right\} =$$
$$= F_1'(\beta_c, S_{2n}) + C_4 i \sqrt{\frac{\Gamma}{a_2}} \left[F_2(\beta_c, S_{2n}) + \beta_c F_2'(\beta_c, S_{2n}) \right].$$

The solution to (19) is put as a series in accordance with the above arguments:

$$T_1 = T_{c_1} + \sum_{n=1}^{\infty} C_{1n} (v_0 + \Gamma x)^{\frac{S_{1n}}{\Gamma}} \left[F_1(y, S_{1n}) + C_{2n} i y \sqrt{\frac{\Gamma}{a_1}} F_2(y, S_{1n}) \right],$$
$$T_2 = T_{c_2} + \sum_{n=1}^{\infty} C_{3n} (v_0 + \Gamma x)^{\frac{S_{2n}}{\Gamma}} \left[F_1(y, S_{2n}) + C_{4n} i y \sqrt{\frac{\Gamma}{a_2}} F_2(y, S_{2n}) \right]. \tag{27}$$

The quantities C_{1n}–C_{4n} are defined by (24), (20), (25), and (21), while the eigenvalues are derived from (26).

The method can be extended to the drawing of a multilayer film.

If the elastic parameters of the components are known as functions of temperature, one can use (27) to optimize the production of crimped materials [8].

NOTATION

v_0, v_f, initial and final velocities; δ_0, initial thickness of bicomponent film; δ_1, δ_2, initial thicknesses of the first and second components; L, drawing zone length; β_1, β_2, ordinates of the surfaces of the components; β_c, ordinate of interface; T_{c_1}, T_{c_2}, temperatures at surfaces of the components; T_1, T_2, temperature distributions in the components; a_1, a_2, thermal diffusivities; x, y, coordinates; $\psi_1(y)$, $\psi_2(y)$, initial temperature distributions; λ_1, λ_2, thermal conductivities; α_1, α_2, heat-transfer coefficients for the outer surfaces; t_1, t_2, excess temperatures.

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